CBCS SCHEME

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17CS36

Third Semester B.E. Degree Examination, Jan./Feb.2021
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 100

Note: ***Answer any FIVE full questions, choosing ONE full question from each module.***

Module-1

1 a. Prove that $(p \vee q) \wedge \neg(\neg p \wedge q) \Leftrightarrow p$, using laws of logic. **(06 Marks)**

b. Establish the following argument by the method of proof by contradiction:
$[(p \rightarrow (q \wedge r)) \wedge (r \rightarrow s) \wedge (\neg(q \wedge s))] \rightarrow \neg p$ **(07 Marks)**

c. Negate and simplify : (i) $\forall x, [p(x) \rightarrow \neg q(x)]$ (ii) $\exists x, [\{p(x) \vee q(x)\} \rightarrow r(x)]$ **(07 Marks)**

OR

2 a. Verify that $[p \rightarrow (q \rightarrow r)] \rightarrow [(p \rightarrow q) \rightarrow (p \rightarrow r)]$ is a tautology. **(06 Marks)**

b. Test the validity of the argument :
"Rita is baking a cake. If Rita is baking a cake, then she is not practicing her flute. If Rita is not practicing her flute, then her father will not buy her a car. Therefore Rita's father will not buy her a car. **(07 Marks)**

c. Prove that for all real numbers x and y, If $x + y > 100$, then $x > 50$ or $y > 50$ by direct proof and contradiction proof. **(07 Marks)**

Module-2

3 a. Prove that for every positive integer n, 5 divides $n^5 - n$. **(06 Marks)**

b. Total of Rs 1500 is to be distributed to three students A, B and C. In how many ways the distribution can be made in multiple of Rs 100.
(i) If each gets at least Rs 300.
(ii) If A must gets at least Rs 500, B and C get at least Rs 400 each? **(07 Marks)**

c. Find the coefficient of $a^2b^3c^2d^5$ in the expansion of $(a+2b-3c+2d+5)^{16}$. **(07 Marks)**

OR

4 a. Let $a_0 = 1$, $a_1 = 2$, $a_2 = 3$ and $a_n = a_{n-1} + a_{n-2} + a_{n-3}$, for all positive integer $n \geq 3$. Then prove that $a_n \leq 3^n$ for all positive integer n. **(06 Marks)**

b. How many ways can 10 oranges be distributed among five children if, (i) there are no restrictions (ii) each child gets at least one (iii) the oldest child gets at least two oranges. **(07 Marks)**

c. Determine the number of integer solutions of a + b + c + d = 32, where
(i) a, b, c and d > 0 (ii) a, b ≥ 5 and c, d ≥ 7. **(07 Marks)**

Module-3

5 a. Define one-to-one function and onto function with example. Determine whether or not the relation $\{(x, y)/x, y \in R$ and $x = y^2\}$ is a function. **(06 Marks)**

b. Let $A = \{a, b, c, d, e\}$ and the relation $R = \{(a,a), (a,e), (b,c), (b,d), (c,c)(d,c), (e,d)(e,a)\}$, write the relation matrix and digraph of R. **(07 Marks)**

c. Draw the Hasse diagram for the subset relation on the power set of $A = \{a, b, c\}$. **(07 Marks)**

OR

6 a. For any sets A, B, C $\subseteq$ U, prove that $A \times (B - C) = (A \times B) - (A \times C)$. **(06 Marks)**

b. Show that if any (n + 1) numbers from {1, 2, 3…..2n} are chosen, then two of them will have their sum equal to (2n + 1). **(07 Marks)**

c. Let $A = \{1, 2, 3, 4, 5\}$ and R be a relation on $A \times A$ defined by (a, b) R (c, d) if and only if a + d = b + c. Show that R is equivalence relation. Determine the partition induced by R and the equivalence class [(2, 5)]. **(07 Marks)**

Module-4

7 a. Determine the number of positive integers n where $1 \leq n \leq 100$, and n is not divisible by 2, 3 or 5. **(06 Marks)**

b. An apple, a banana, a mango and an orange are to be distributed for four boys A, B, C, D. The boys A and B do not wish to have apple. C does not want banana or mango and D refuses orange. In how many ways the distribution can be made so that no boy is displeased? **(07 Marks)**

c. Solve $2a_n = 7a_{n-1} - 3a_{n-2}$ for $n \geq 2$ and $a_0 = 2$, $a_1 = 5$. **(07 Marks)**

OR

8 a. Define the principle of inclusion and exclusion and generalization of the principle. **(06 Marks)**

b. Find the rook polynomial of the chess board. (Refer Fig. Q8 (b)). **(07 Marks)**

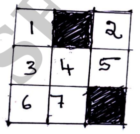

Fig. Q8 (b)

c. Solve $a_n = 2a_{n-1} - 2a_{n-2}$ for $n \geq 2$ and $a_0 = 1$, $a_1 = 2$. **(07 Marks)**

Module-5

9 a. Discuss the Konigsberg-bridge problem and solution. **(06 Marks)**

b. Define Isomorphic graphs. Show that the following two graphs are isomorphic. (Refer Fig. Q9 (b)) **(07 Marks)**

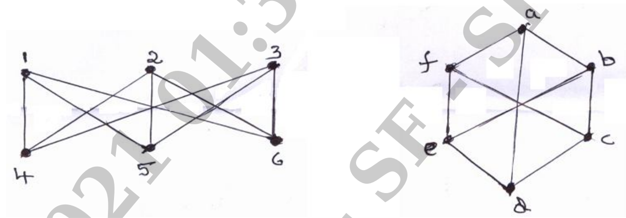

Fig. Q9 (b)

c. Construct an optimal prefix code for the symbols a, b, c, d, e, f, that occur with respective frequencies 20, 28, 4, 17, 12, 7. **(07 Marks)**

OR

10 a. Find the number of spanning sub graphs of the graph given below. How many of them are connected. How many are spanning trees? **(06 Marks)**

b. Prove that for every tree $T \equiv (V, E)$, if $|V| \geq 2$ then T has at least two pendent vertices. **(07 Marks)**

c. Define directed tree, rooted tree, binary rooted tree, complete binary tree, m-ary tree complete m-ary tree, leaf. **(07 Marks)**

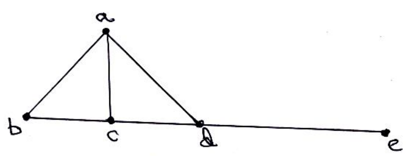

Fig. Q10 (a)